Starter

In the binomial expansion of
where ,
the coefficient of is 4 times
the coefficient of .

Find the value of .

$$\square\square\square = \frac{\square!}{\square!\left(\square-\square\right)!}$$

0 Statistical hypothesis testing

01

Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value; extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded).

Students should:

- recognise whether a given context requires the use of a 1-tail or 2-tail test and understand the difference between them
- be able to state appropriate null and alternative hypotheses to test a population proportion in a given context and know that the null hypothesis always contains the equality sign
- understand that the significance level of a test is the probability of rejecting a correct null hypothesis in error
- be able to find the test statistic, which is the observed number of outcomes of the event
- be able to find the critical region for a 1-tail test, or the critical regions for a 2-tail test, supporting the choice of values in such regions with appropriate binomial probabilities
- know that the critical region consists of the critical values for the test and that if the test statistic lies in the critical region then the null hypothesis is rejected
- know that the acceptance region is the range of possible values, that the discrete random variable can take, that do not lie in the critical region and that if the test statistic lies in the acceptance region that this will lead to the acceptance of the null hypothesis
- be able to use the given p-value corresponding to the test statistic or the given critical value(s), for the relevant significance level of the test, to decide whether to accept or reject the null hypothesis; understand that the p-value should be compared to a binomial distribution critical region with probability equal to or less than the significance level
- know that the precise definition of a p-value in a 2-tailed test varies. It can be defined as the probability calculated from the test statistic or twice that value. In order to circumvent this difficulty, questions will not be asked in which students are required to state the p-value for such a test
- be able to interpret a conclusion in context.

0 Statistical hypothesis testing

Notes

- The conclusion of a hypothesis test is an inference based on evidence and thus students must indicate that there is no certainty in their conclusions. Using the phrase “sufficient evidence to suggest” (qualified with a “not” as applicable) would be a good standard to adopt. There is nothing to be gained by trying to write a conclusion creatively. The final concluding statement of a hypothesis test should always relate back to the context.
- In cases where the null hypothesis is not rejected we would allow an inference of “Do not reject H0” or “Accept H0.” Statistical purists will prefer the former.

11.1 Hypothesis Testing

General Method

1. Define the population parameter in context (Let be…).
2. State the null hypothesis ().
3. State the alternative hypothesis ().
4. State the test statistic (Let be the number of…).
5. Write the probability distribution of (Under ,).
6. *State the significance level ().*
7. *Test for significance (-value) OR find the critical region.*
8. *Write conclusion in context (is there sufficient evidence to reject .)*

11.1 Hypothesis Testing

Once we have set up the test (from last lesson) we now need to decide whether our observed value from a sample provides enough evidence to allow us to reject the null hypothesis or not.

i.e. is the probability of the ‘number of successes’ that we observed in our sample so unlikely to happen that we can reject this claim?

11.1 Hypothesis Testing

We must first know the **significance level**, , that we are going to compare our probability to (this will be stated in the question).

The significance level of a test is the probability of rejecting a correct null hypothesis in error. Smaller values of will reduce the chance of making such an error.

e.g. in drug trials, a much smaller significance level would be required.

11.1 Hypothesis Testing

The significance level of a test determines how unlikely your data needs to be under the null hypothesis before we reject H_0.

e.g. = 0.05 (5%) would mean we **only** reject H_0 if the observed data fell into the **most extreme** 5% of all possible outcomes.

11.1 Hypothesis Testing

To test an observed value for significance we must work out its **p-value** – the probability of X being **at least as extreme** as this observed value, assuming H_0 is true. We find this probability using the Binomial CD function.

We compare this **p-value** to the **significance level** and write our **conclusion** (in context). If the p-value is less than the significance level we reject H_0.

11.1 Hypothesis Testing

Example 3

Ed believes that a five-sided spinner is biased towards landing on a 5. He spins the spinner 20 times and it lands on a 5 ten times. Using a 5% level of significance, test the hypothesis that the spinner is biased towards landing on a 5.

p is the probability that the spinner lands on a 5.

H_0: $p = 0.2$

H_1: $p > 0.2$ (one-tailed)

X is the number of times it lands on a 5 in the sample.

Under H_0, $X \sim B(20, 0.2)$

11.1 Hypothesis Testing

<u>Example 3</u>

p is the probability that the spinner lands on a 5.

H_0: p = 0.2

H_1: p > 0.2 (one-tailed)

X is the number of times it lands on a 5 in the sample.

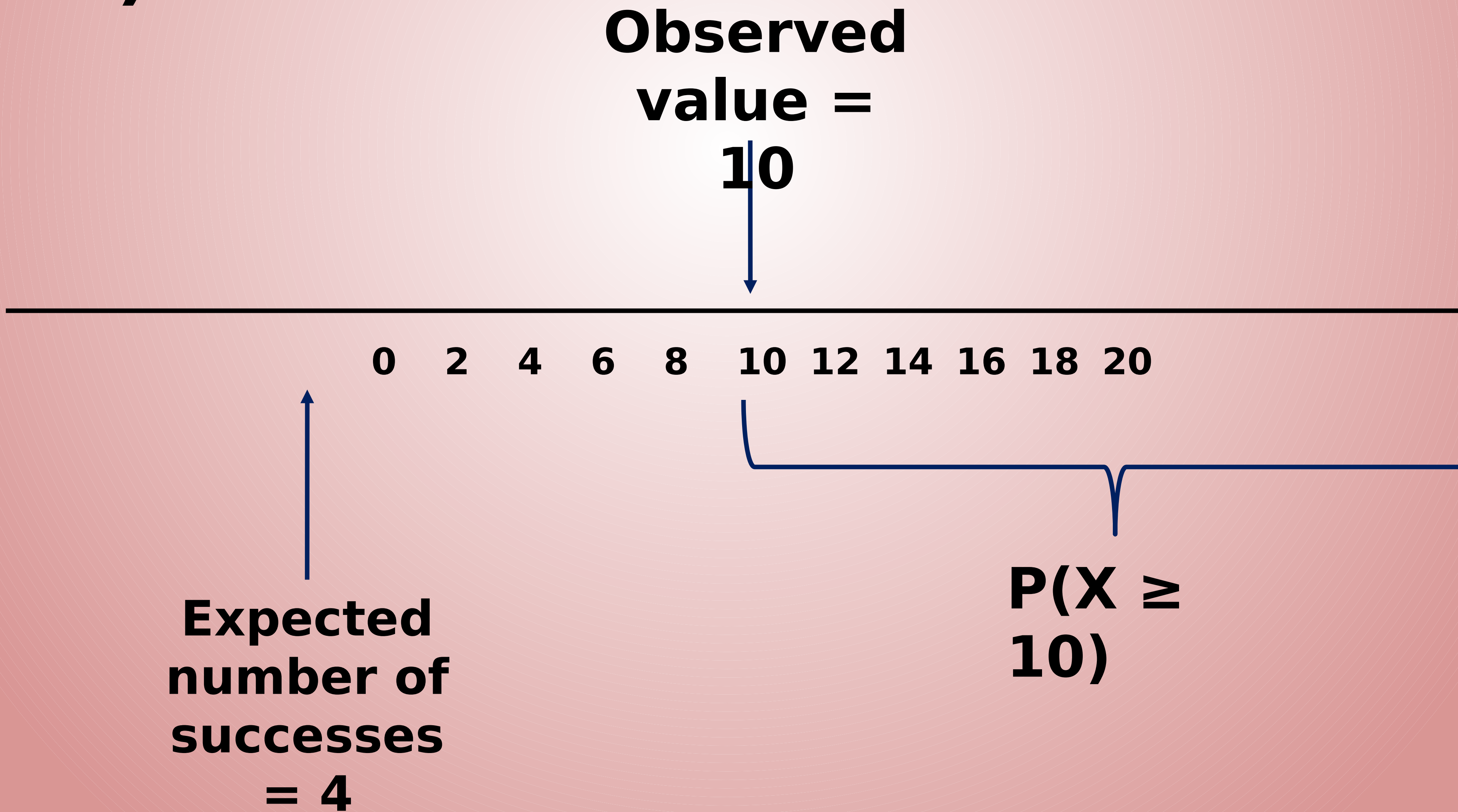
Under H_0, X ~ B(20, 0.2)

= 0.05 *[5% significance level]*

P(X ≥ 10) = 1 – P(X ≤ 9) = 0.003 (3dp) *[p-value]*

Since 0.003 < 0.05 the result is significant. We

Why do we use P(X ≥ 10)?



11.1 Hypothesis Testing

Example 4

The standard treatment for a particular disease has a $\frac{2}{5}$ probability of success. A certain doctor has undertaken research in this area and has produced a new drug which has been successful with 11 out of 20 patients. The doctor claims that the new drug represents an improvement on the standard treatment. Test, at the 5% significance level, the claim made by the doctor.

Let p be the probability of the new drug being successful.

Let X be the number of successful treatments in the sample.

Under

Not significant. Do **not** reject .

There is **not** sufficient evidence to suggest that the new drug is more

11.1 Hypothesis Testing

Example 5

A dice used in playing a board game is suspected of not giving the number 6 often enough. During a particular game it was rolled 12 times and only one 6 appeared. Does this represent significant evidence, at the 5% level of significance, that the probability of a 6 on this dice is less than $\frac{1}{6}$?

Let p be the probability of rolling a 6.

Let X be the number of sixes rolled in the sample.
Under

Not significant. Do not reject .
There is not sufficient evidence to suggest that the dice lands on a 6 less than expected.

11.1 Hypothesis Testing

General Method

1. Define the population parameter in context (Let be…).
2. State the null hypothesis ().
3. State the alternative hypothesis ().
4. State the test statistic (Let be the number of…).
5. Write the probability distribution of (Under ,).
6. State the significance level ().
7. Test for significance (-value) OR find the critical region.
8. Write conclusion in context (is there sufficient evidence to reject .)

Exercise 2.2